

Introduction

This book has as its focus the important topic of the teaching of number. It describes a teaching approach and related teaching activities that have been developed in the Mathematics Recovery Programme and the Count Me In Too projects in Australia, the UK, the USA and Canada. The central thrust is the teaching of number skills, knowledge and understanding which is present in the number strand of a primary mathematics curriculum.

The authors have a particular concern for those aspects of early number taught in the 4- to 8-year age range. The reason for this is that research studies in the 1990s showed that there are significant differences in the numerical knowledge of children when they begin school (Aubrey, 1993; Wright, 1991a; 1994; Young-Loveridge, 1989; 1991). These studies also show that these differences in number knowledge increase as children progress through schooling. There is a clear tendency for low attainers in the early years to continue to be low attainers throughout their primary years and to develop negative attitudes to mathematics. There is a need therefore to give every child a positive understanding and success in early number work.

But how does one achieve that? Let us look at the teachers' experiences from the three areas involved in the Mathematics Recovery and Count Me In Too projects. Whether in Australia, the UK or the USA we contend that teachers have a similar basic set of questions to address when planning teaching. We would suggest that the following are prominent in teachers' thoughts:

- ▶ What knowledge does the child possess?
- ▶ What are the child's current misunderstandings and misconceptions?
- ▶ Where do I want to take the child?
- ▶ Given that the child is having difficulty with 'x', which is a necessary part of learning 'y', what are the optimum tasks, examples and settings that will allow the child to progress?
- ▶ What specific materials will I use?
- ▶ How can I establish the linkages and generalities in the learning, which will allow progress to be made?
- ▶ When I provide these experiences what will I look for to be able to gather evidence that the child is learning?
- ▶ What possible misconceptions might occur and how can I remediate these?

We would contend that the teacher also has other questions to address. A particularly difficult one relates to one's own confidence level in understanding the mathematical content. It is easy to state but hard to confront. It is 'Do I understand the mathematics involved?' Given the nature and content of early numeracy this should not be hard but let us rephrase the question to read: 'First, do I understand in which order I should teach the content and second, how do the aspects of what I am teaching fit together and lead to a greater understanding of the operations in numeracy?' Internationally, many countries and states have focussed energy and resources on the review of the mathematics curriculum, the refinement and articulation of attainment targets for each age group, the production of curriculum materials and resource books for teachers, and the attendant professional development training courses. Consequently, teachers have much clearer guidelines and there has been a growth in teacher confidence and children's attainment.

However, we would contend that not everything is as straightforward as it may seem. The guidance produced suggests targets to be met across year groups. Furthermore suggested, even mandated, styles of teaching have put a greater emphasis on whole-class interaction in the introductory and plenary parts of the numeracy lesson. Even when group work is evident, government reports indicate that children are not being sufficiently challenged. The activities in the teachers' handbooks may be interesting and fun for the children but do they engender learning which relates to the individual child's needs?

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Where then, is the help to be given to the teacher of numeracy in the early years who has a class with a wide range of abilities and a significant number of children already beginning to demonstrate low attainment? We think it is a valid question to ask that under pressure to advance a designated proportion of children to a certain level of achievement, what time, help and support can be given to the children who are beginning to fall behind?

WHAT IS SPECIAL ABOUT THIS BOOK?

In writing this book we want to address this problem and provide practical help for the teacher. It is not a book to dip into for interesting exercises, though we provide plenty of these. The authors consider that this book differentiates itself from other books on numeracy because we present a comprehensive and integrated approach to assessment, learning and teaching which is strong on providing teachers with a clear direction and purpose based on three sound principles.

First, we consider that it is crucially important for teaching to take full account of the child's current numerical knowledge and strategies used to solve problems. The term 'strategies' is used here to refer to procedures that children might use to solve an addition task such as $8 + 4$ or a division task such as 'Twelve biscuits are divided between some children. Each child gets two biscuits each. How many children received the biscuits?' The term 'numerical knowledge' includes aspects such as identifying written symbols for numbers (for example, 7, 35) and knowing sequences of number words such as 'one, two, three ...'.

Second, we present a learning framework for number, that is, a framework that links closely to the assessment tasks and sets out children's learning pathways for teachers. An example of the framework in action can be seen when a young child is asked to solve $5 + 4$. The child says the forward number word sequence 'one, two, three, four, five' and raises the fingers on one hand sequentially in coordination with the number words. The child continues 'one, two, three, four' and raises four fingers on the other hand sequentially. The child then counts the raised fingers from one, 'one, two, three, four, five, six, seven, eight, nine, ... Nine!' The above could well be accompanied by touching the face with each raised finger in turn.

In the example above the child used a relatively low level strategy which we describe as 'counting from one'. The Learning Framework in Number, which is explained in full in Chapter 1, provides detailed guidance on how the teacher can advance the child so that the use of finger patterns becomes more sophisticated or redundant and the child now routinely uses a more advanced strategy.

Thirdly, we present a set of principles and procedures for teaching that are closely linked with the learning framework. We demonstrate how a child can progress from one level of strategy to the next more advanced level, by illustrating the learning framework in action using six key topics, each of which spawns a multitude of practical and motivating teaching activities.

WHAT IS THE BACKGROUND TO THIS BOOK?

The approach presented is drawn from a programme of numeracy research and development projects which have been implemented widely in schools in several countries since the early 1990s. Many of these implementations have taken place under the label of Mathematics Recovery. The Mathematics Recovery Programme involves an extensive study of school-based and team-based professional development. It aims to advance teachers' knowledge of the assessment, learning and teaching of early number.

The initial development of Mathematics Recovery occurred over a three-year period (1992–95) in the Australian state of New South Wales. The development was funded by the Australian Research Council and participating school systems. Since 1995, the Mathematics Recovery Programme has

been widely implemented in 24 states in the USA, and in the Bahamas and Canada. In total this has involved approximately 350 specialist teachers, 60 leaders of training and more than 3,000 participating children. In England and Wales 21 education authorities have implemented Mathematics Recovery on a sustainable basis and in Scotland and Ireland over 200 teachers at all levels have been trained in assessment and intervention.

Furthermore, in a recent comprehensive study on the help available to children with numeracy difficulties for the Department of Education and Skills for England and Wales (Dowker, 2004), the Mathematics Recovery Programme was identified as one of only two large-scale independently developed, individualized intervention programmes which specifically address numeracy difficulties in young children. The research report indicated how Mathematics Recovery targeted the younger primary age group, 4–8-year-olds, drew together research findings regarding the stages of development and, most importantly, made them applicable to teaching and teachers of early numeracy. The report highlighted how children in the programme showed significant improvement even after short interventions of between 10 and 12 hours' duration and how the children reached, or even exceeded, the age-related norms for their classmates. The report also indicated that teachers saw their work in the Mathematics Recovery Programme as being a very important part of their professional development, enabling them to be more confident in diagnosing children's difficulties in early numeracy, to design and implement effective interventions and to advise colleagues on courses of action.

In 1996 the Mathematics Recovery Programme was adapted by the state department of education in New South Wales as the basis of a systemic initiative in mathematics in the early years of schooling and was called Count Me In Too (CMIT). In its initial year, CMIT was piloted in 13 schools. In subsequent years (1997 to 2004) CMIT was progressively implemented in virtually all of the 1,700 schools (primary/elementary) across the state. In addition, CMIT has been widely adopted by school systems in other Australian states, and in 2000 CMIT was the basis of a nationally funded pilot project in New Zealand involving 81 schools (Thomas and Ward, 2001). This pilot of CMIT, together with the Mathematics Recovery Programme, informed the New Zealand Numeracy Development Project which was implemented nationally from 2001 onward (Bobis et al., 2005).

A list of selected publications relating to both the Mathematics Recovery Programme and Count Me In Too is included in the Bibliography.

PURPOSE OF THIS BOOK

In the revised edition of *Early Numeracy: Assessment for Teaching and Intervention* (Sage 2006), the authors set out a detailed approach to the assessment of children's early numerical strategies and knowledge. The book provides six diagnostic interview schedules for focussing on a range of aspects of early number. The schedules are administered via videotaped assessment interviews and used to elicit children's strategies and number knowledge. As well, the book sets out procedures for analyzing the results of the assessment interviews, and a framework for determining a child's strategies and documenting current levels of a child's knowledge. The framework just referred to is called the Learning Framework in Number (LFIN).

The approach to assessment just described has been used by a range of school systems in at least five countries, as part of the Mathematics Recovery project or programmes based on the Count Me In Too initiative. This work has involved several thousand teachers and many thousands of children. The current authors and their colleagues have spent countless hours conducting school-based and system-based professional development meetings which have focussed specifically on this approach to assessment, and more generally on ways of observing and interpreting children's mathematical activity and thinking. In these meetings we are frequently asked by teachers and leaders to provide advice

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about teaching approaches that accord with children's mathematical thinking. Many of the questions in this vein relate to a specific episode or event which the participants have all just observed via a videotaped replay of assessment or teaching, because videotaped records of assessment and teaching sessions constitute an important means by which to observe practice during the professional development meetings.

One might respond to questions of the kind just described in general terms by referring the audience to the LFIN, in which case the teachers who ask the question (that is, about teaching) would be expected to refer to the framework and attempt to determine appropriate instructional approaches. For many of the teachers with whom we have worked this kind of approach has worked well. In Mathematics Recovery for example, teachers are provided with an extensive bank of instructional settings and activities, as well as videotapes exemplifying the use of these. Another way to respond when asked what teaching is appropriate given a particular episode or event in an assessment or teaching session is to select or describe a setting and demonstrate teaching procedures and learning activities. Again, this approach has been used frequently by the current authors in Mathematics Recovery professional development meetings.

The purpose of this book is to answer systematically the questions described above, that is, questions relating to the need for teaching to be informed by assessment and to accord with the LFIN. The book provides detailed descriptions of specific instructional settings, teaching procedures and learning activities. These descriptions are organized in terms of the LFIN and relate closely to the outcomes of the assessment procedures set out in the related books *Early Numeracy: Assessment for Teaching and Intervention* (Sage 2000) and *Early Numeracy: Assessment for Teaching and Intervention, Second Edition*, (Sage 2006).

THE STRUCTURE OF THE BOOK

In this introduction we have described the scope of this book, that is, the teaching of early number and we have outlined what is special about this book. We have also introduced the reader briefly to two interrelated initiatives which have had a significant impact internationally in the area of early number learning. These are the Mathematics Recovery Programme and the New South Wales Count Me In Too early numeracy initiative. Further, we have provided an international overview of recent issues and developments in mathematics in the early years of school. Finally, we have referred to our previous book which sets out a comprehensive approach to assessment in early number and describes a comprehensive learning framework which guides assessment and provides a means of observing and documenting the outcomes of assessment.

Chapter 1 consists of an introductory section and two main sections. In the introductory section we describe how we use terms such as 'strategies' and 'knowledge' in describing children's early number learning. In the first main section we describe the Learning Framework in Number (LFIN) which provides essential guidance for teaching as well as for assessing. This section includes links from the LFIN to key topics and teaching procedures in Chapters 5–9. The second main section describes aspects of our approach to teaching early number.

From Chapter 2 onwards the reader is provided with a comprehensive guide to teaching early number using the LFIN. This guide includes a focus on individualized teaching as it might be used, for example, in an intervention programme, as well as a focus on classroom teaching. The purpose of Chapter 2 is to provide for the reader, a detailed overview of significant aspects of the approach to individualized teaching which has been developed in the Mathematics Recovery Programme. These aspects are organized into the following three sections: Section A – the guiding principles of individualized teaching; Section B – key elements of individualized teaching; and Section C – characteristics of

children's problem-solving in individualized teaching sessions. The final section of Chapter 2 consists of four scenarios of children's learning in individualized teaching sessions. These scenarios serve to exemplify the aspects described in Sections A, B and C.

In Chapter 3 key attributes of whole-class teaching of number are discussed. A variety of lesson formats is described. The chapter first describes the fundamental instructional goals of sense-making and intellectual autonomy. The chapter then focusses on the teaching and learning cycle which is described in four sections: 'Where are they now?', 'Where do you want them to be?', 'How will they get there?' and 'How will you know when they've arrived?'

Chapter 4 provides a detailed overview of the common format that is used in Chapters 5–9. This includes an overview of the 30 key instructional topics that are presented in Chapters 5–9, that is, six in each chapter. Each key topic consists of around six teaching procedures. In this way, each of the five chapters includes at least 32 teaching procedures. The description of each teaching procedure includes examples of a teacher's words, actions, and notes on purpose, teaching and children's responses.

Chapters 5–9 set out a detailed framework for teaching which relates closely to and is informed by the LFIN. Each of these five chapters focusses on teaching children at a given stage in terms of the Stages of Early Arithmetical Learning. In this vein Chapter 5 focusses on teaching children at the Emergent Stage, Chapter 6 focusses on the Perceptual Stage, Chapter 7 on the Figurative Stage, Chapter 8 on the Stage of Counting-On and Down-From, and the Stage of Counting-Down-To, and Chapter 9 on the Facile Stage. As well, each of these stages assumes specific levels of knowledge in relation to other aspects of the LFIN, namely, forward number word sequences (FNWSs), backward number word sequences (BNWSs), numeral identification and tens and ones.

Chapters 5–9 have a common structure. First, an overview is provided of the early number knowledge and strategies typical of children at the stage and levels specified for that chapter. This is followed by detailed outlines of six key teaching topics considered likely to be significant for advancing the knowledge of such children. The key topics relate to a range of important aspects of the LFIN, and each outline of a key topic includes descriptions of, on average, six teaching procedures relevant to that key topic. Instructional materials, key vocabulary and links to the LFIN are included for each key topic, and each teaching procedure includes notes on purpose, teaching and children's responses. Each of Chapters 5–9 also includes outlines of several lessons which exemplify whole-class teaching to children assumed to be at the stage and levels specified for that chapter.

Across Chapters 5–9 there is a total of 182 teaching procedures. This very large corpus of teaching procedures, each of which can be understood in terms of the LFIN, is a particularly distinctive feature of this book. The authors' intention is that the reader should regard the teaching procedures as illustrative and should adopt and adapt procedures as they see fit. The procedures are not intended to be followed verbatim. To do so would not accord with current practice in Mathematics Recovery teaching sessions, nor would it accord with the guiding principles of individualized teaching set out in Chapter 2 of this book.